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Weak-disorder effects in the finite-charge infinite-U Hubbard model

W-N Huang and J W Rasul
Physics Department, University of Michigan, Ann Arbor, MI 48109-1120, USA

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Abstract. We calculate the effects of weak disorder on a strongly correlated electron system, namely a version of the infinite-U Hubbard model with general charge for which a $1/N$ expansion can be carried out (N being the orbital degeneracy). The partition function and quasiparticle decay rate are calculated to next leading order in the $1/N$ expansion by summing all ladder type diffusive corrections in the impurity potential. We find that away from the critical metal–insulator filling these quantities exhibit the power law behaviour expected on the basis of general weak-interacting theories. Very close to the metal–insulator filling, however, the situation changes and new, sublinear power law components are obtained. These new power laws are a result of the diffusion pole crossing over to a q^4 behaviour, which is in turn a consequence of the 'holon' like propagation of the charge fluctuations in the pure system.

1. Introduction

The extraordinary sensitivity of strongly correlated electron systems to small amounts of impurity provides a challenging new aspect to the theory of interacting disordered systems. Experimentally, heavy-fermion systems show huge changes in the residual resistivity virtually regardless of whether the dilution is taking place on the normal host or the rare earth sites [1]. In high-temperature superconductors a small amount of impurity can drive a transition directly from the superconducting to the insulating phase [2].

From the theoretical point of view, most theoretical effort has started from the disordered weakly localized system, and incorporated the effects of electron interactions in the diffusive regime [3]. More recently scaling approaches have been applied to the problem [4], but from a microscopic point of view, the aim has basically been to incorporate interaction effects into the very thorough field theoretical treatment that describes the Anderson transition [5].

Most recently this idea has been applied by Zimanyi and Abrahams [6] to the t–J model. These authors used the now prevalent auxiliary boson representation to enforce the restricted hopping condition. They combined this with a mean field *ansatz* for these boson amplitudes that reflected the strong underlying site disorder, by coupling these real boson amplitudes to the local disorder via the charge susceptibility. The resulting model then consists of a random hopping term together with an attractive interaction between the fermions (from the antiferromagnetic coupling term). This interaction was treated via the renormalization group techniques of Finkelshtein [7].

An alternative approach is to begin directly from the strongly correlated electron system and assess the effects of increasing the disorder. For heavy-fermion systems a number of studies have taken this approach [8–10], starting with the large-N limit of the infinite-U Anderson lattice (N being the orbital degeneracy), in which the infinite-U restriction is

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handled by introducing Bose fields [11, 12] that modulate the hybridization and enforce the restriction of the occupation to values between zero and unity. In the large-N limit these Bose fields are replaced by their saddle point values (to be determined self-consistently) with the result that the effective Hamiltonian at large degeneracy consists of hybridizing states of conduction and local electrons, the latter being located a small energy above the Fermi surface. The real saddle point values of the Bose fields are responsible for this renormalization.

In these approaches [8–10] the disorder is then introduced by adding site disorder terms diagonal either in the f states or in the Wannier representation of the conduction states, or by adding disorder to the hybridization. Using either the self-consistent Born approximation [8] or the coherent potential approximation (CPA) [9] the effects of disorder are absorbed into self-energy corrections to the conduction and local propagators. The heavy-fermion states near the Fermi level are broadened by the disorder, through the self-energy acquiring a finite imaginary part, while the saddle point values of the Bose fields are still found to be real, as in the pure case.

Of these approaches the first, by Tesanovich [8], studied the combined effects of disorder among both the local and band states by calculating the Born approximation to the electrons' self-energy matrix, ensuring that the same self-energy appeared in the intermediate electronic state. The disorder average was then performed over the mean field equations for the Bose fields, by replacing the intermediate electronic Green functions by their disorder averaged Born approximation expressions. While the occupation constraint field was found to be largely unaffected by the disorder, the hybridization field was found to decrease slowly at first with disorder, and then sharply disappear as a critical value of the disorder width (of the order of the Kondo temperature) was approached. This disappearance was interpreted as the destruction of coherence, and the transition to a dense Kondo state.

This work was followed by a CPA calculation by Xu and Li [9] who showed that diluting the f states did not lead to a vanishing of the hybridization field—in fact they found this field to be hardly affected as a function of disorder. The self-energy corrections obtained using the CPA did however lead to a disappearance of the hybridized band structure and its replacement, in the spectral function, by a form characteristic of the dilute Kondo state. They also subsequently calculated the resistivity ρ and found it to follow a $\rho_0 + AT^2$ law with the coefficient A changing from negative to positive as the f-state concentration increased. Freytag and Keller [9] used the same CPA method to calculate the dynamical conductivity. They again found mean field parameters unchanged from their values in the pure system. The dynamic conductivity was shown to have two peaks, the first being a Drude peak, followed by a second peak due to interband transitions.

However, it is well known from long experience with the disordered electron problem that the subtle effects associated with the metal–insulator transition require going beyond the CPA and including the effects of quantum interference or weak-localization corrections. One of the present authors [13] carried out such a calculation for the case of potential scattering acting on the conduction electrons in the Anderson lattice, summing up the maximally crossed or Langer–Neal corrections to the electrical conductivity. It was found that frequency dependent effects such as the AC conductivity suffered a large renormalization of the frequency scale consistent with the heavy-fermion mass enhancement while the static conductivity was described by the standard weak-localization expression [3], i.e. without any of the unique effects associated with the heavy-fermion state.

All of these auxiliary boson approaches [6–13] are at the mean field level—our intention here is to focus on one-loop, Gaussian $1/N$ corrections. It is known that for the pure Anderson lattice model [12] these corrections take a form akin to the random phase

approximation (RPA) used standardly in weak-interacting-electron theories. It is also well known that even weak disorder has a profound effect on such RPA corrections in weak-interaction theories [3]. The RPA corrections to bulk properties are affected, acquiring novel power law temperature dependent corrections to quantities such as the specific heat. These corrections arise from ladder type dressings of the RPA bubbles, which enter the free energy. To obtain such corrections it is necessary to include only the lowest-order Born approximation correction to the electron self-energy—the combination of this together with the ladder terms in the impurity potential lead to diffusive behaviour in the density correlation function.

The difficulty with the Anderson lattice lies in the two-band nature of the problem. It is difficult to include all the possible sources of disorder—on the f states, the conduction states or the hybridization. As a precursor to treating the Anderson model it is helpful to consider a simpler one-band model that can be formulated along the same lines and whose mean field theory embodies similar physical principles as in the Anderson lattice. Such a model is the infinite-U Hubbard model with finite electron charge [14–16]. If the charge is scaled by the degeneracy then this model, at the mean field level, experiences a mass enhancement akin to the Gutzwiller approximation. This results from the occupation restriction imposed by the $U = \infty$ constraint. The mass enhancement is proportional to the fraction of vacant sites. In the pure case the $1/N$ [15] and $1/N^2$ [17] corrections to the quasiparticle scattering amplitudes have been investigated.

For disordered one-band models such as the Hubbard model the approach described above has not been carried out. In the present paper we consider the N-fold degenerate infinite-U Hubbard model in the presence of weak site disorder, using the $1/N$ expansion to treat the strong correlations, and including the effects of weak disorder by (i) treating the self-energy to the Born approximation in the impurity potential and (ii) summing up those ladder graphs in the impurity potential that usually yield diffusive behaviour in the density correlation function. We expect on the one hand to make contact with conventional 'weakly interacting' theories [3] if the electron filling in the $U = \infty$ Hubbard model is not too close to $1/N$, the critical value at which the insulating phase takes over. We hope in this way to understand the effects of Gutzwiller projection on conventional disorder ideas.

Another reason underlying our investigation of this problem concerns the behaviour close to $1/N$ filling. In the pure system the present authors [16] found that the nature of the collective mode in the density fluctuation channel changes as this filling fraction is approached. Instead of the acoustic 'zero-sound' behaviour the collective mode has a dispersion functionally identical to that of the bare electron motion in the model—if the electron bands are parabolic then so is the collective mode; if the electron band has rather a tight-binding form then this is also reflected in the collective mode. It is important to notice that the bare electron mass, not the Gutzwiller renormalized mass, enters this dispersion relation—the electrons can move as a whole in this way avoiding the infinite-U restriction. Independently Wang *et al* [18] found such a collective mode in the t–J model, which they ascribed to 'holon' motion.

The present authors investigated the contribution of this mode at second order in $1/N$ to the quasiparticle interaction vertex [17] and found a rather substantial contribution, considerably larger than the leading order $1/N$ results [15]. In this paper we shall investigate the nature of this mode when substantial disorder is present—we shall see that the usual diffusion pole behaviour is completely overwhelmed in the region close to $1/N$ filling by a q^4 term. We shall see that this leads to a completely different behaviour in the low-temperature specific heat and quasiparticle decay rate to the conventional disordered interacting result.

The plan of the paper is as follows: in the next section we present the formulation of the $U = \infty$ Hubbard model in the presence of disorder, deriving the mean field equations and the Bose fluctuation propagators. In the following sections we discuss the results for the density correlation function, the specific heat corrections and the on-shell quasiparticle decay rate.

2. Formulation

Our staring point is the infinite-U Hubbard model for which the Lagrangian can be written

$$L(\tau) = \sum_{i,j,m} f^+_{i,m}(\tau)\{[\partial/\partial\tau - \mu + i\lambda_j(\tau)]\delta_{ij} + t_{ij}\rho_i\rho_j\}f_{jm}(\tau) + \sum_j i\lambda_j\left(\rho_j^2 + \sum_m f^+_{im}f_{im} - Q\right) \tag{1}$$

where f_{im} denotes a fermion operator at site i and in orbital m, μ denotes the chemical potential, λ_i denotes a Lagrange multiplier, originally introduced to enforce the infinite-U constraint, and ρ_i denotes the radial part of the Bose field that labels the empty site at site i. The quantity Q is usually allowed to scale with the degeneracy $N = \sum_n$, and is taken to be unity at the end of the calculation. The standard gauge transformation [19] has been performed so that the Lagrange multiplier is now time dependent. To the above Lagrangian we add the following terms in the case of site disorder:

$$L_{\text{imp}}(\tau) = \sum_{R_a, r_i} u(r_i - R_a) f^+_{im}(\tau) f_{im}(\tau) \tag{2}$$

where R_a denotes the set of impurity site positions each of which exert a potential $u(r_i - R_a)$ on electron at site r_i. The partition function is written in terms of the above as

$$Z = \int Df^+ Df\, D\rho\, D\lambda\, \exp\left(-\int_0^\beta d\tau\, [L(\tau) + L_{\text{imp}}(\tau)]\right) \tag{3}$$

where the site disorder term is absorbed into the constraint term $i\lambda_j(\tau) = i\lambda_j(\tau) + \sum_{R_a} u(r_j - R_a)$.

Integrating (3) over the fermion fields yields the effective action

$$L(\tau) = \sum_i i\lambda_i(\rho_i^2 - Q) - N\text{Tr}\ln\left[\left(\frac{\partial}{\partial\tau} - \mu + i\lambda_j\right)\delta_{ij} + t_{ij}\rho_i\rho_j\right]. \tag{4}$$

Following the standard $1/N$ expansion method we separate the effective action into a mean field component with mean field parameters given, after minimizing the free energy, by

$$i\lambda_0 = -N\text{Tr}\{t_{ij}/[(\partial/\partial\tau - \mu + i\lambda_i)\delta_{ij} + \rho_0^2 t_{ij}]\} \tag{5}$$

$$\rho_0^2 = Q - N\text{Tr}\{1/[(\partial/\partial\tau - \mu + i\lambda_i)\delta_{ij} + \rho_0^2 t_{ij}]\} \tag{6}$$

where the trace is over lattice sites (and the inverse imaginary time derivative can be transformed into a frequency sum) and a Gaussian fluctuation term

$$L_{\text{Gauss}} = \frac{1}{2\beta}\sum_\omega [i\lambda_i i\lambda_m S^{\lambda\lambda}_{im} + i\lambda_m(\omega)\rho_i(\omega)S^{\lambda\rho}_{im} + \rho_m\rho_i(\omega)S^{\rho\rho}_{im}] \tag{7}$$

where the Gaussian order Bose propagators are given by

$$S^{\lambda\lambda}_{im}(\omega_q) = \frac{N}{2\beta}\sum_{\omega_a} G_{im}(\omega_a)G_{im}(\omega_a+\omega_q) \tag{8}$$

$$S^{\lambda\rho}_{im}(\omega_q) = \frac{N}{2\beta}\sum_{\omega_a jp}[G_{im}(\omega_a)G_{mp}(\omega_a+\omega_q)\rho t_{pi} + G_{pm}(\omega_a)G_{mi}(\omega_a+\omega_q)\rho t_{ip}] + \rho \tag{9}$$

$$\begin{aligned} S^{\rho\rho}_{im}(\omega_q) = {} & \frac{N\rho}{2\beta}\sum_{\omega_a}[G_{im}(\omega_a)G_{np}(\omega_a+\omega_q)t_{mn}t_{pi} + G_{sm}(\omega_a)G_{ni}(\omega_a+\omega_q)t_{mn}t_{is} \\ & + G_{in}(\omega_a)G_{mp}(\omega_a+\omega_q)t_{mn}t_{pi} + G_{sn}(\omega_a)G_{mi}(\omega_a+\omega_q)t_{nm}t_{is}] \\ & + i\lambda_0 + \frac{N}{\beta}\sum_{\omega_a} G_{im}(\omega_a)t_{im}. \end{aligned} \tag{10}$$

The $G_{ij}(\omega_a)$ denote the mean field propagators, whose traces enter into (5) and (6). So far, the impurity positions are still explicitly specified, and the configurational averages have yet to be carried out. Formally the next step is to perform the disorder average, which we carry out by performing the Gaussian integral over the Bose fields and expanding the resulting free energy in a power series in the RPA bubbles:

$$F = \frac{1}{2\beta}\sum_{\omega_q}\mathrm{Tr}\ln(\Pi_0+\Pi_{ij}) = \frac{1}{2\beta}\sum_{\omega_q}\mathrm{Tr}[\ln\Pi_0] + \sum_n \frac{[(\Pi_0)^{-1}\Pi_{ij}]^n}{n} \tag{11}$$

where the Π_{ij} refer to the RPA components of the Bose propagators in (11). Following the standard approach to calculating the free energy in an interacting disordered system [3] we replace the disorder average of Π^n_{ij} by $\langle\Pi_{ij}\rangle^n$ in order to extract the dominant low-energy diffusive corrections to the RPA bubbles. These are obtained in the standard manner by summing the ladder diagrams in the impurity potential (see figure 1). The free energy then becomes

$$F = (1/2\beta)\mathrm{Tr}\ln(\Pi_0 + \langle\Pi_{ij}\rangle) \tag{12}$$

and the Bose propagators have translational invariance restored and take the form

$$\begin{aligned} S^{\rho\rho}(q,\omega_q) = {} & i\lambda - \frac{N}{\beta}\sum_{\omega_q} t_{k+q}G^{ex}(k,\omega_a) + N\frac{\rho^2}{2\beta} \\ & \times\sum_{\omega_a}[I_2(q,\omega_a,\omega_q) + n_i u(0)^2 I_1(q,\omega_a,\omega_q)^2 S(q,\omega_a,\omega_q)] \end{aligned} \tag{13}$$

$$S^{\lambda\rho}(q,\omega_q) = i\rho\left(1 + \frac{N}{2\beta}\sum_{\omega_a} I_1(q,\omega_a,\omega_q)[1 + n_i u(0)^2 I_0(q,\omega_a,\omega_q)S(q,\omega_a,\omega_q)\right) \tag{14}$$

$$S^{\lambda\lambda}(q,\omega_q) = \frac{N}{2\beta}\sum_{\omega_a} I_0(q,\omega_a,\omega_q)[1 + n_i u(0)^2 I_0(q,\omega_a,\omega_q)S(q,\omega_a,\omega_q)] \tag{15}$$

where G^{ex} denotes the exact quasiparticle Green function appearing at the leading order in $1/N$, averaged over the disorder. The I_n are given by

$$I_n(q,\omega_a,\omega_q) = \sum_k (t_k + t_{k+k_1})^n G(k_1,\omega_a)G(k+k_1,\omega_a+\omega_q) \tag{16}$$

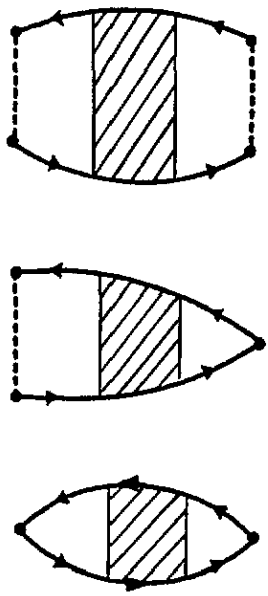

Figure 1. Ladder diagrams in the impurity potential that contribute to the RPA components of the Bose propagators. The full curves denote the mean field electron Green functions with self-energies given by the Born approximation in the impurity potential. The shaded areas denote the results of summing ladder diagrams to infinite order in the impurity potential, and correspond to the quantity $S(q, \omega_a, \omega_q)$ defined in (17). The dots where two mean field Green functions meet denote a factor of unity, while dots joined by a broken line denote a hopping matrix element. In this way all the RPA components of the Bose propagators (in descending order) $S_{\rho\rho}$, $S_{\rho\lambda}$ and $S_{\lambda\lambda}$ are obtained. A similar diagram can be drawn for the quantity $L_{ijkl}(E, \omega)$ that enters into the decay rate calculation.

and the quantity $S(q, \omega_a, \omega_q)$ is defined by

$$S(q, \omega_a, \omega_q) = \left(1 - \frac{1}{2\tau N^*(0)} \sum_k G(k+q, \omega_a + \omega_q) G(k, \omega_a)\right)^{-1}. \tag{17}$$

The impurity effects are taken into account both in the ladder sum entering the above expression and in the self-energy corrections to the electron Green function:

$$G(k, \omega_a)^{-1} = i\omega_a - \rho^2 t_k (i/2\tau) \operatorname{sgn} \omega_a \tag{18}$$

where, as in the conventional weak-disorder approaches, we keep only the lowest-order Born approximation to the impurity self-energy correction to the Green functions that enter into the ladder sum. The density of states at the Fermi level is given by $N^*(0)$ and the quasiparticle lifetime is given by

$$1/2\tau = n_i u(0)^2 N^*(0) \tag{19}$$

where $u(0)$ denotes the zero-wavevector component of the impurity potential. At this stage it is convenient to address the effects of disorder on the mean field quantities ρ and $i\lambda$. The self-energy broadening of the spectral function given by (19) allows the mean field equations to be transformed to a simple form in which the frequency sums can be carried out with the result that

$$i\lambda = -N \sum_k t_k [\tfrac{1}{2} - \tan^{-1}(2\tau\varepsilon_k)] \tag{20}$$

$$\rho^2 = Q - N \sum_k [\tfrac{1}{2} - \tan^{-1}(2\tau\varepsilon_k)]. \tag{21}$$

The only difference between these mean field equations and those in the pure case is that the Fermi function is broadened by the impurity scattering. The mean field values of the

Bose fields are still real. Since the saddle point values of the Bose fields require integrations over wavevector k it is clear that these saddle point values suffer corrections of the order of $(\varepsilon_F\tau)^{-1}$. Such corrections are analytic in the disorder. In the weak-disorder limit we may neglect these and focus only on the $1/N$ fluctuation corrections, which are far more serious. We note that in the CPA treatments of the disordered Anderson lattice the mean field Bose parameters are also only weakly affected by the disorder [9].

Returning to the Bose propagators we find that the ladder sum term $S(q, \omega_a, \omega_q)$ may be evaluated for small ω_q and q to give

$$S(q, \omega_a, \omega_q) = [\theta(\omega_a)\theta(-\omega_a - \omega_q) + \theta(-\omega_a)\theta(\omega_a + \omega_q)]/(|\omega_q|\tau + Dq^2\tau). \tag{22}$$

Here D denotes the diffusion constant $v_F^2\tau/d$ where d is the dimensionality and v_F the renormalized Fermi velocity. The low-wavevector and low-energy limits of the I_n are also required:

$$I_n(0, 0, 0) = 2\pi\tau N^*(0)(2t_0)^n. \tag{23}$$

After some manipulation we obtain the following expressions for the Bose propagators at low q, ω:

$$S^{\rho\rho}(q, \omega_q) = -N\frac{q^2}{2m\beta}\sum_{\omega_a, k_a} G(k_a, \omega_a) + (2t_0\rho)^2 F(q, \omega_q) \tag{24}$$

$$S^{\lambda\rho}(q, \omega_q) = \rho[1 + 2t_0 F(q, \omega_q)] \tag{25}$$

$$S^{\lambda\lambda}(q, \omega_q) = F(q, \omega_q) = -NN^*(0)Dq^2/(2|\omega_q| + 2Dq^2) \tag{26}$$

which defines $F(q, \omega_q)$. Here t_0 denotes the bare kinetic energy at the Fermi surface and we have used the mean field results to simplify the first two terms in $S^{\rho\rho}$ in the following manner: we have expanded t_{k+q} in q (assuming a parabolic dispersion) in the second term in (13) and made use of the first mean field equation ((5) or (20)) to cancel part of this t_{k+q} term with the first term on the RHS of (13). This leaves a cross term between k and q, which, after integrating over angle, vanishes by symmetry, and a q^2 term. This q^2 term (the first term in (24)) will later be rewritten using the second mean field equation (averaged over disorder) in terms of the band filling.

3. Charge susceptibility and specific heat

Turning to the physical consequences of the above we note that the addition of the source field term

$$L_{\text{source}} = \int_0^\beta \sum_i \alpha_i(\tau) \sum_m f_{im}^+(\tau) f_{im}(\tau) \tag{27}$$

allows us to derive in standard fashion [16] the charge susceptibility

$$\chi(q, \omega) = \rho^2 S_{\lambda\lambda}(q, \omega)/\det S(q, \omega) \tag{28}$$

which has a pole at the frequency ω_q defined as the zero of

$$\det S(q, \omega_q) = S^{\lambda\rho}(q, \omega_q)^2 - S^{\lambda\lambda}(q, \omega_q)S^{\rho\rho}(q, \omega_q). \tag{29}$$

On using the small-ω, small-q expansion for the boson propagator given in equations (24)–(26), we obtain

$$\chi(q,\omega_q) = F(\omega_q,q)\Big/\left[1+4\left(t_0-\frac{q^2}{2m}\frac{N}{\beta}\sum_{k_a\omega_a}G(k_a,\omega_a)\right)F(q,\omega_q)\right] \tag{30}$$

with $F(\omega_q,q)$ given by (26).

We note at this point the formal similarity between this result and standard RPA type expressions, bearing in mind that $F(\omega_q,q)$ represents a particle–hole bubble dressed by ladder graphs in the impurity potential. If the impurity potential were absent then $F(\omega_q,q)$ would be equal to the Gutzwiller enhanced Lindhard function and (30) would be identical to the charge susceptibility of the $U=\infty$ model obtained before [16]. For moderate fillings the charge susceptibility of the pure system shows a zero-sound pole with a sound velocity dependent on the zero-q limit of the effective interaction (the term in round brackets in the denominator of (30)).

However, as is obvious from (30) this effective interaction also contains a q^2 term with a coefficient which, on using the mean field equation (6) can be written αq^2 where $\alpha=\frac{3}{4}(Q/N)^2(m^*/m)^2$ and Q is the total charge per site defined earlier. The important feature of this q^2 term is its strong dependence on the mass enhancement, which itself diverges as the metal–insulator filling is approached. This results, for fillings close to the critical value, in the zero-sound mode crossing over to a q^2 value (for a parabolic band) at arbitrarily small q values. Wang *et al* [18] obtained such a collective mode in the t–J model, which they interpreted as 'holon' motion—the electrons prefer to move as a whole, thus avoiding the infinite-U constraint. The present authors showed that for general electron dispersions, the collective mode had the same dispersion as the electron band.

Returning to the disordered system, we examine the result of making these substitutions in the charge susceptibility, which then takes the form

$$\chi(q,\omega_q) = Dq^2NN^*(\varepsilon_F)/2(|\omega_q| + Dq^2A_0 + D\alpha q^4/k_F^2) \tag{31}$$

where $A_0 = 1-2t_0NN^*(\varepsilon_F)$.

This expression shows, after analytic continuation of ω_q to the complex plane, a pole off the real axis which, at sufficiently low q, behaves like a diffusion pole with a modified diffusion constant A_0D. This is in agreement with weakly interacting theories (with short-range interactions) where the diffusion constant is modified by an interaction parameter. However, as in the case of the pure system the importance of the wavevector dependent part of the interaction (the q^2 term) grows as the metal–insulator filling is approached. Because of the fact that α is proportional to the square of the mass enhancement, we expect the q^4 term in the denominator of (31) to dominate the q^2 term over an increasing wavevector region as the metal–insulator filling is approached. Hence for fillings arbitrarily close to the critical value the pole in the charge susceptibility will follow a Dq^4/k_F^2 law.

A question naturally arises at this point as to the magnitude of this q^4 contribution relative to the standard q^4 corrections to the diffusion pole that would normally arise in a non-interacting system, i.e. that we would obtain when we expand the ladder series result for the non-interacting charge susceptibility to next order in q^2. This is easily explored by expanding the denominator in (17), after which we obtain the dominant contribution to be

$$q^4\frac{n_iu(0)^2}{(2m)^4}\sum_p\frac{p^4\cos\theta^4}{(\varepsilon_p+i/2\tau)^5(\varepsilon_p-i/2\tau)} \tag{32}$$

which, after converting the wavevector integral to one over the energy, setting $p = \varepsilon_F$ in the numerator, and rescaling the energy by $1/\tau$, is easily shown to be of the order of $(v_F\tau q)^4$ (where v_F denotes the Fermi velocity). This results in corrections to the denominator of $F(\omega_q, q)$ of the order of $D(q^4/k_F^2)(\varepsilon_F\tau)^2$ so that the condition for this to be smaller than the q^4 correction arising from the wavevector dependence of the interaction (which is of the order of $Dq^4/k_F^2(m^*/m)^2$) is that $m^*/m \gg \varepsilon_F\tau$. While we naturally require $\varepsilon_F\tau$ to be large for our weak-disorder approach to be valid, we also have the filling (and hence m^*/m) as an adjustable parameter, so that this condition can be achieved.

Thus for fillings arbitrarily close to the metal–insulator filling the q^4 term arising from the wavevector dependence of the quasiparticle interaction will dominate over the usual diffusive q^4 terms and also over the q^2 terms for a region of q such that $q > k_F m/m^*$. Asymptotically close to the metal–insulator filling the diffusion propagator will be proportional to $Dq^2/i\omega - D(m^*/m)^2(q^4/k_F^2)$.

Particularly interesting consequences of the present theory concern the low-temperature specific heat, which in the weakly interacting disorder theories is expected to show novel temperature dependences, depending on the dimensionality [3]. In contrast non-interacting systems show localization effects only in their transport properties.

The free energy at Gaussian $1/N$ order can then be written

$$F = \frac{1}{2\beta}\sum_{\omega,q} \ln\det S(q,\omega) = \frac{-1}{2\pi}\sum_q \int d\omega\, n(\omega) \times \left[\tan^{-1}\left(\frac{\omega}{ADq^2 + \alpha Dq^4/k_F^2}\right) - \tan^{-1}\left(\frac{\omega}{Dq^2}\right)\right] \tag{33}$$

where we have replaced the sum by an integral over the Bose function $n(\omega)$ and again used the small-ω, small-q form of the Bose propagators. This form is not adequate for anything other than the leading non-analytic corrections to the Fermi liquid results—in a pure system such an approach yields the $T^3 \ln T$ behaviour in the specific heat. It is particularly convenient to take one temperature derivative of the free energy so that

$$\frac{\partial F}{\partial T} = \frac{-2}{\pi}\sum_q \int_0^\infty dx\, e^x x \left[\tan^{-1}\left(\frac{x}{f(q,T)}\right) - \tan^{-1}\left(x\frac{T}{Dq^2}\right)\right] \Big/ (e^x - 1)^2 \tag{34}$$

where $f(q,T) = (ADq^2 + \alpha Dq^4/k_F^2)/T$. We next perform the q integral, ignoring the terms linear in temperature that require a momentum cut-off and focus instead on the non-analytic parts for which the remaining integrals are easily evaluated.

For the case of moderate mass enhancements we may neglect the q^4 terms and obtain for the total specific heat

$$C_V = p_3(1 - A^{-3/2})(T/D)^{3/2} \tag{35}$$

in three dimensions and

$$C_V = p_2(1 - 1/A)(T/D)\ln(T/D) \tag{36}$$

in two dimensions. Here the constants are given by $p_2 = (1/4\pi^2)a(2) = \frac{1}{12}$, $p_3 = (1/2\pi^3)a(\frac{5}{2})b(0,4)$ with $a(m)$ and $b(m,n)$ defined by

$$a(m) = \int_0^\infty x^m \frac{e^x}{(e^x - 1)^2}\, dx \tag{37}$$

$$b(m,n) = \int_0^\infty \frac{x^m}{1 + x^n}\, dx. \tag{38}$$

We may compare these terms with the results obtained in weakly interacting disorder theories [3]—the result of Altshuler and Aronov (equation (355) of their review) is that for three dimensions the specific heat correction may be written (using our definitions (37) and (38))

$$C_V = p_3\{1 - 3[(1 + F/2)^{3/2} - 1]\}(T/D)^{3/2} \tag{39}$$

while in two dimensions the Altshuler–Aronov result can be written

$$C_V = p_2(1 - 3F/2)T/D \ln(T\tau) \tag{40}$$

where F denotes the static $q = 0$ limit of the interaction between spin-one electron–hole pairs in a general interacting electron system. The extra constants in (39) and (40) (the non-F terms) arise from the effective interactions in the spin-zero particle–hole channel. The reason that the spin-zero (density fluctuation) terms do not explicitly depend on a Landau parameter is that in the spin-zero channel Altshuler and Aronov [3] included the long-range Coulomb interaction. Since this diverges at long wavelengths it swamps the remaining short-range Landau parameters that are finite as $q \to 0$. After including Debye screening effects the resulting interaction in the spin-zero channel is universal, i.e. independent of Landau parameters—hence the constants in (39) and (40).

Consequently in such general weakly interacting theories only the spin-one terms (for which no intermediate Coulomb propagator can be included) depend on the static Landau parameters. In addition they obtain a factor of three from the spin multiplicity. In our model, being a Hubbard model with the corresponding assumption of perfect screening and the absence of long-range Coulomb interaction, (39) and (40) constitute the large-N counterpart of the spin-zero results of Altshuler and Aronov [3] (with the omission of the long-range term). In view of the absence of the long-range term the results do depend on the microscopic details of the model, which in the case of $U = \infty$ is the electronic filling factor n.

We note the absence of the spin-one terms in our result. This is a general feature of large-N theories at the one-loop level. Only density correlations are included at the $1/N$ level. This arises from the fact that the bosons in (1) couple to the total fermion density, not the spin density. A similar feature emerges at the $1/N$ level for the Anderson lattice—the $T^3 \ln T$ component of the specific heat only includes density fluctuations [12].

However, as far as the overall power law dependences are concerned we find complete agreement between our results away from the metal–insulator transition and the earlier weak-interaction theories [3]. The situation changes as we approach the $1/N$ fractional filling limit. This time the q^2 terms in the denominator of the first term in (33) are swamped by the q^4 terms with the result that the specific heat now acquires a contribution

$$C_V = \int_0^\infty x\,\mathrm{d}x \frac{\mathrm{e}^x}{(\mathrm{e}^x - 1)^2} \sum_q \tan^{-1}\left(\frac{xTk_F^2}{D\alpha q^4}\right). \tag{41}$$

Thus the low-temperature behaviour is now controlled by the superdiffusive mode in the density correlation function with the result that the temperature scaling of the wavevector in (41) is now half of the power that appears in the normal diffusive case. Hence we obtain new temperature dependences in the specific heat as a result of this term. Evaluating the resulting integrals we obtain

$$C_V = q_3(T/D\alpha)^{3/4} + p_3(T/D)^{3/2} \tag{42}$$

in three dimensions where $q_3 = (1/2\pi^3)a(\frac{7}{4})b(6, 8)$ and

$$C_V = q_2(T/\alpha D)^{1/2} + p_2 T \ln(T/D) \tag{43}$$

in two dimensions where $q_2 = (1/2\pi^2)a(\frac{3}{2})b(5, 8)$. Thus at sufficiently low temperatures the specific heat develops a sublinear fractional power law dependence, which may even dominate over the normal linear specific heat term, however weak the disorder. This tendency towards destroying the metallic character of the system is specific to the strong-interaction case—it runs counter to the weak-interaction folklore. Moreover it implies that arbitrarily weak disorder can substantially modify the metallic ground state even in three dimensions as long as the system is sufficiently near the Mott transition. We note that the result near $1/N$ filling in two dimensions has the same functional dependence on temperature as weakly interacting theories give in one dimension, again emphasizing the destabilizing of the Fermi liquid that the combination of strong interaction and weak disorder can lead to.

As regards the crossover between these two kinds of power law, we can estimate this temperature by studying the first term in (34). The most rapid variation in the q integral occurs where the argument of the inverse tangent (and hence $f(q, T)$) becomes of the order of unity. If this happens in the crossover region then we require that the q^2 and q^4 terms in $f(q, T)$ are equal, and that the temperature is also of this order. From this it follows that the crossover temperature is of the order of $\varepsilon_F(\varepsilon_F\tau)A_0^2(m/m^*)^2$, which may be made arbitrarily small compared with the effective bandwidth ε_F if the electron filling is taken to be close to its critical metal–insulator value. Below this temperature (39) and (40) take over from (42) and (43).

To conclude this section, we have studied the effects of weak disorder on thermodynamic properties of the $U = \infty$ Hubbard model, using the $1/N$ expansion to treat the strong correlation effects, and summing ladder graphs in the impurity potential to obtain localization effects. For fillings away from the critical value, the usual diffusion pole behaviour is found in the density correlation function, with the infinite-U effects reflected in a Gutzwiller type mass enhancement. The $1/N$ contribution to the specific heat shows the same temperature dependence as found in previous weak-interaction approaches. However, as the filling approaches its critical value, the diffusion pole behaviour is modified, the q^2 term being replaced by a q^4 contribution. The specific heat at the $1/N$ level now shows novel sublinear temperature dependences ($T^{3/4}$ and $T^{1/2}$ in three dimensions and two dimensions) over an increasing temperature region as the critical filling is approached.

4. The quasiparticle scattering rate in the presence of disorder

In the previous section we examined the effects of the combination of strong electron–electron interactions and weak disorder on the static bulk properties of a system. From an experimental point of view one would prefer to be able to probe directly aspects of the single-particle Green function that are known to be uniquely affected by the simultaneous presence of interaction and disorder.

The aspect of the one-electron properties that has most in common with the specific heat is, not surprisingly, the self-energy. From conventional Fermi liquid theory in pure systems one knows that a $T^3 \ln T$ term in the specific heat at low temperature is related to an $\varepsilon^3 \ln \varepsilon$ contribution to the real part of the self-energy. This can in turn, by a Kramers–Kronig relation, be linked to an ε^3 term in the imaginary part of the self-energy. Such a feature is a well known aspect of interacting translationally invariant systems. Likewise, for our

disordered problem, there is a connection between the behaviour found in the imaginary part of the self-energy and the low-temperature specific heat. For weakly interacting disordered systems, it was shown by Altshuler and Aronov [20] that these components can be simply related, much along Landau theory lines, in terms of the same set of parameters. In our strongly interacting problem, we hope to find a similar link between the novel power laws seen close to half-filling in the specific heat and the energy dependence of the quasiparticle decay rate. We also have the added convenience that by going to lower electron fillings, we should be able to make contact with the weak-interaction theories, and relate the 'Landau' parameters obtained by Altshuler and Aronov to the microscopic model parameters of our problem (the band filling).

Having explained our motivation for studying the quasiparticle decay rate, we encounter an immediate problem as to its calculation. In a pure system the lowest-order contribution to the decay involves the scattering of a quasiparticle off dynamic bosonic charge density or spin density fluctuations of the system. This gives rise to the ε^2 and ε^3 components of the quasiparticle lifetime. In our problem, however, we are faced with the issue that we do not actually know how to write down a Green function for the quasiparticle, since we do not know the eigenstates of the disordered electron problem. Even though we do know the form of the bosonic density fluctuation spectrum, namely the diffusion pole behaviour, we cannot simply convolute this against the lowest-order Green function

$$G(k,\omega) = 1/(\mathrm{i}\omega - \varepsilon_k - \mathrm{i}/2\tau) \tag{44}$$

since in the limit $\varepsilon_F^*\tau \gg 1$, this leads to a quasiparticle lifetime of the order of $(\varepsilon_F^*\tau)^{-1}$. Essentially this boils down to the fact that within the context of a Green function such as (1) with such a large elastic scattering rate, concepts such as self-energy and decay rate are meaningless. This caused serious problems for many-body approaches to the problem. In fact the earliest correct results [21, 22] were obtained directly from the Boltzmann equation. The way out of this was shown by Abrahams *et al* [23], who pointed out that even for a weak static two-body interaction potential, one had to deal with the exact eigenstates $\Psi_m(r)$ and creation operators a_m^+ of the disordered one-electron problem. The interaction term then becomes

$$H = \int \mathrm{d}r \int \mathrm{d}r' \sum_{m,n,p,q} \Psi_m^*(r')\Psi_n^*(r)v(r-r')\Psi_p(r)\Psi_q(r')a_m^+a_n^+a_pa_q \tag{45}$$

with a self-energy given to leading order by

$$\Sigma_m = \sum_n \int \mathrm{d}r\,\mathrm{d}r'\Psi_m^*\Psi_n^*\Psi_m\Psi_n v(r-r'). \tag{46}$$

To obtain a meaningful scattering rate we have to average this over the electron states

$$\Sigma(E) = \sum_m \langle \delta(E-E_m)\Sigma_m\rangle \tag{47}$$

which requires that we calculate the quantity

$$\sum_{m,n} \langle \delta(E-E_m)\Psi_m^*\Psi_n^*\Psi_m\Psi_n\rangle \tag{48}$$

which is itself related to the two-particle Green function of the disordered interacting electron system. Thus even the lowest-order Hartree–Fock self-energy requires a knowledge of

the density–density correlation function. When we allow scattering of the full diffusive propagator, we shall then obtain a convolution between two density–density correlation functions. Since our $1/N$ method separates out the non-interacting, disordered quasiparticles appearing at the mean field level from the diffusive density correlations that appear at the $1/N$ level, we shall adopt the method of Abrahams *et al* [23].

We proceed therefore to investigate the effect of diffusive density fluctuations on the lifetime of the exact eigenstates of the disorder problem. Our approach is to introduce the exact eigenstates of the disorder $U = \infty$ mean field problem, and calculate the scattering rate of these quasiparticles from the boson density fluctuations (which have the diffusion pole behaviour at moderate fillings and a 'super-diffusive' q^4 behaviour at fillings close to $1/N$). We introduce a one-electron set of eigenstates (which we shall define later) by

$$f_{im}^{+} = \sum_{\alpha} a_{\alpha,m}^{+} \Psi_{\alpha}^{*}(i). \tag{49}$$

Then the partition function is given by

$$Z = \int \mathrm{D}a^{+}\,\mathrm{D}a\,\mathrm{D}\rho\,\mathrm{D}\lambda\,\exp\left(\int_0^{\beta} \mathrm{d}\tau L(\tau)\right) \tag{50}$$

where

$$L(\tau) = \sum_i \mathrm{i}\lambda_i(\tau)[\rho_i^2(\tau) - Q] - N\mathrm{Tr}\ln$$
$$\times \sum_{ij} \Psi_{\alpha}^{*}(i)\left[\left(\frac{\partial}{\partial\tau} - \mu + \mathrm{i}\tilde{\lambda}_i(\tau)\right)\delta_{ij} + t_{ij}\rho_i(\tau)\rho_j(\tau)\right]\Psi_{\beta}(j) \tag{51}$$

where $\mathrm{i}\tilde{\lambda}_i(\tau) = \mathrm{i}\lambda_i(\tau) + \sum_{R_a} u(r_i - R_a)$, as before, incorporates the effect of the random site disorder into the on site Lagrange multiplier. We expand, in the standard $1/N$ manner, the fermionic part of the Lagrangian in powers of the fluctuating Bose fields; this then becomes

$$A_{\alpha\beta}(\mathrm{i}\omega_a, \mathrm{i}\omega_b) = \beta A_{\alpha\beta}^{0}(\mathrm{i}\omega_a) + X_{\alpha\beta}(a, b) \tag{52}$$

where

$$A_{\alpha\beta}^{0}(\mathrm{i}\omega_a) = \sum_{ij} \Psi_{\alpha}^{*}(i)\left[\left(\mathrm{i}\omega_a - \mu + \mathrm{i}\lambda_0 + \sum_{R_a} u(r_i - R_a)\right)\delta_{ij} + t_{ij}\rho_0^2\right]\Psi_{\beta}(j) \tag{53}$$

and

$$X_{\alpha\beta}(a, b) = B_{\alpha\beta}(a, b) + C_{\alpha\beta}(a, b) + D_{\alpha\beta}(a, b) \tag{54}$$

where

$$B_{\alpha\beta}(a, b) = \sum_{ij} \Psi_{\alpha}^{*}(i)\mathrm{i}\lambda_i(\omega_a - \omega_b)\Psi_{\beta}(j)\delta_{ij} \tag{55}$$

$$C_{\alpha\beta}(a, b) = \sum_{ij} \Psi_{\alpha}^{*}(i)t_{ij}\rho[\rho_i(\omega_a - \omega_b) + \rho_j(\omega_a - \omega_b)]\Psi_{\beta}(j) \tag{56}$$

$$D_{\alpha\beta}(a, b) = \sum_{ij} \Psi_{\alpha}^{*}(i)t_{ij}\frac{1}{\beta}\sum_{\omega_q} \rho_i(\omega_a - \omega_b - \omega_q)\rho_j(\omega_a)\Psi_{\beta}(j) \tag{57}$$

are the fluctuating components to first and second order in the Bose fields. The eigenstates $\Psi_\alpha(i)$ are now defined as those states that diagonalize the Gutzwiller mass enhanced Hamiltonian that emerges, albeit in the presence of site disorder, at leading order in $1/N$. Turning now to the actual Green function we note that it is usual to study the gauge invariant correlation function

$$\langle b_i(\tau) f^+_{im}(\tau) f_{jm}(\tau_1) b^+_j(\tau_1)\rangle = G_{ij}(\tau - \tau_1) \tag{58}$$

which on Fourier transforming, and using the expansion (6) in terms of exact eigenstates of the disorder problem, becomes to leading $1/N$ order

$$G^0_{ij}(\mathrm{i}\omega) = \sum_\alpha \Psi_\alpha(i)\Psi^*_\alpha(j)\rho_0^2 A^{-1}_{0\alpha}(\mathrm{i}\omega) \tag{59}$$

showing the presence of a quasiparticle weight factor $\rho_0^2 = 1 - n$ as well as the exact (to leading $1/N$ order) single quasiparticle Green function. Corrections at $1/N$ order come from a number of different sources, and in order to be precise about what we mean by a scattering rate we have to examine these in turn. Consider the most general form expected for an interacting Green function near its quasiparticle pole

$$G_\alpha(\mathrm{i}\omega) = Z_\alpha(\mathrm{i}\omega)/[\mathrm{i}\omega - \varepsilon_\alpha - \Sigma_\alpha(\mathrm{i}\omega)] + G^{\mathrm{inc}}(\mathrm{i}\omega) \tag{60}$$

where, as is evident from the present calculation, Σ_α and the incoherent term G^{inc} are all of the order of $1/N$, while Z_α has, in addition, a leading order term ρ_0^2. We turn therefore to the boson fluctuation corrections to (58), and start by performing the expansion of the fermion propagator $A^{-1}(\omega_a, \omega_b)$ to leading order in $1/N$. Firstly expanding both the Bose fields in (58) to Gaussian order and carrying out the functional integral over the Bose fields yields a term

$$\sum_\alpha \Psi_\alpha(i)\Psi^*_\alpha(j)\rho_0^2 \frac{1}{\beta}\sum_{\omega_1} S^{-1}_{\rho\rho}(i, j, \omega_1) A^{-1}_{0\alpha}(\mathrm{i}\omega + \mathrm{i}\omega_1) \tag{61}$$

where $S_{\rho\rho}$ denotes a boson propagator of the type examined earlier. We have, in keeping with the rest of this paper, averaged the Bose fluctuations separately from the fermion Green function. This contribution leads, from our previous discussion after equation (58), to a $1/N$-order incoherent part to the spectrum. This is straightforward to see, since in fact it has precisely that form of a self-energy that we discussed in the introduction. Since it gives no information on the quasiparticle pole that might help understand the quasiparticle lifetime, we ignore it. Similarly, keeping one of the boson fluctuation factors in (58), together with the lowest-order term in the expansion of A^{-1} (from (54)) leads to a contribution to $G_{ij}(\mathrm{i}\omega)$ equal to

$$\rho_0 \frac{1}{\beta}\sum_{\omega_n} G^0_{ij_1}(\mathrm{i}\omega) S^{-1}_{\rho\lambda}(j, j_1, \mathrm{i}\omega_2) G^0_{j_1 j}(\mathrm{i}\omega - \mathrm{i}\omega_2) \tag{62}$$

with G^0 as defined in (59). Again, comparing this with the $1/N$ expression of the Green function discussed earier we see that (60) has the form of a spatial convolution of G^0 with a self-energy correction akin to (61). This expression therefore can be regarded as renormalizing the quasiparticle weight $Z(\mathrm{i}\omega)$, and therefore does not affect the quasiparticle

lifetime. Thus we see that the only $1/N$ contributions to the quasiparticle are those given by replacing the boson operators in (56) by their mean field values, and expanding

$$\rho_0^2 \int \mathrm{D}\rho\, \mathrm{D}\lambda\, A_{\alpha\beta}^{-1} \Psi_\alpha(i) \Psi_\beta^*(j) \exp\left(- \int_0^\infty \mathrm{d}\tau L(\tau) \right) \tag{63}$$

to Gaussian order in the Bose fields, which leads to a correction to $G_{ij}(\mathrm{i}\omega)$ equal to

$$\rho_0^2 \sum_\varepsilon \Psi_\varepsilon(i) \Psi_\varepsilon^*(j) A_{\varepsilon\varepsilon}^{-1(0)}(\mathrm{i}\omega_\mathrm{a}) \Sigma_{\varepsilon\varepsilon}(\mathrm{i}\omega_\mathrm{a}) A_{\varepsilon\varepsilon}^{-1(0)}(\mathrm{i}\omega_\mathrm{a}) \tag{64}$$

where the $1/N$ correction to the self-energy is given by

$$\Sigma_{\varepsilon\varepsilon}(\mathrm{i}\omega_\mathrm{a}) = \frac{1}{\beta} \sum_{\gamma,c} X_{\varepsilon\delta}(a,c) A_\delta^{0-1}(c) X_{\delta\varepsilon}(c,a). \tag{65}$$

This term is a sum of a whole set of contributions, each of which is quadratic in Bose fluctuation operators, and quartic in the eigenstates of the leading-order Hamiltonian. The remaining disorder average is performed in an analogous manner to that in [23], by separating out the Bose fluctuations and replacing these by their disorder averages. This leaves a term quartic in the eigenstates in (65), together with the leading $1/N$-order Green function given in (59). This Green function has the same eigenstate label as two of the four eigenstate functions in (65) and we may simply take this combination to be the site dependent propagator.

$$\sum_\alpha \Psi_\delta^*(i) A_\delta^{0-1}(\mathrm{i}\omega) \Psi_\delta(j) = G_{ij}(\mathrm{i}\omega) \tag{66}$$

which represents the leading $1/N$-order propagator for a particular configuration of scattering impurities. Another point to be noted concerns the actual definition of the scattering rate. Since we are unable to calculate the exact large-N Green function for a particular configuration, we can only hope to give a meaning to the impurity averaged self-energy

$$\Sigma_E(\mathrm{i}\omega) = \frac{1}{N^*(0)} \sum_\varepsilon \delta(E - E_\varepsilon) \Sigma_\varepsilon(\mathrm{i}\omega). \tag{67}$$

The delta function combines with the two remaining wavefunctions in such a way as to be simply related to another site dependent large-N Green function

$$\sum_\varepsilon \delta(E - E_\varepsilon) \Psi_\varepsilon^*(i) \Psi_\varepsilon(l) = \frac{1}{2\mathrm{i}\pi} [G_{il}^A(E - \mathrm{i}\delta) - G_{il}^R(E + \mathrm{i}\delta)] \tag{68}$$

and performing the impurity average then requires a knowledge of the quantity

$$L_{ijkc}(E, \omega_c) = (1/2\mathrm{i}\pi) \langle [G_{il}^A(E) - G_{il}^R(E)] G_{kj}(\omega_c) \rangle \tag{69}$$

which itself can be expressed in terms of a set of diffusive ladder diagrams in the impurity potential. The full set of contributions to $\Sigma_E(\mathrm{i}\omega_\mathrm{a})$ is then given by

$$\begin{aligned}
\Sigma_E(\mathrm{i}\omega_\mathrm{a}) = \frac{1}{\beta} \sum_{ik} \sum_{\omega_c} \int \mathrm{D}\rho\, \mathrm{D}\lambda\, \{ & L_{iikk}(E, \omega_c) \mathrm{i}\lambda_i(a - c) \mathrm{i}\lambda_k(c - a) \\
& + [L_{ijkk}(E, \omega_c) + L_{jikk}(E, \omega_c)] t_{ij} \rho \rho_i (a - c) \mathrm{i}\lambda_k(c - a) + [L_{iikl}(E, \omega_c) \\
& + L_{iilk}(E, \omega_c)] t_{kl} \rho \mathrm{i}\lambda_i(a - c) \rho_k(c - a) + [L_{ijkl}(E, \omega_c) + L_{jikl}(E, \omega_c) \\
& + L_{ijlk}(E, \omega_c) + L_{jilk}(E, \omega_c)] t_{ij} t_{kl} \rho^2 \rho_i(a - c) \rho_k(c - a) \} \\
& \times \exp\left(- \int_0^\beta \mathrm{d}\tau\, L(\tau) \right) \Big/ Z.
\end{aligned} \tag{70}$$

The impurity average in the exponential has been performed separately from the impurity average over the four eigenstates. The resulting translational invariance allows us to carry out the Gaussian integrals over the Bose fields, which requires knowledge of the Bose propagators calculated in the previous section. To calculate the average four-eigenstate propagator L_{ijkl}, we also adopt the approach of summing up the diffusive low-energy and low-wavevector contributions. We obtain

$$L_{ijkl}(E, \omega_c) = \mathrm{disc}_z Z_{ijkl}(z, \omega_c) \tag{71}$$

where

$$disc_z f(z) = (1/2i\pi)[f(\varepsilon - i\delta) - f(\varepsilon + i\delta)] \tag{72}$$

and

$$Z_{ijkl}(z, \omega_c) = \sum_{k_a, k_l} G(k_l, \omega_c) G(k_a, z) \exp[ik_a(i-l) + ik_l(k-j)] + \sum_{k_a, k_c, k_k} \exp[i(i-k)k_a + i(k-j)k_k + i(k-l)k_c] P(k_a, k_c, k_k, z, \omega_c) \tag{73}$$

where, in addition,

$$P(k_a, k_c, k_k, z, \omega_c) = G(k_a, z) G(k_c, z) n v^2 S(k_a - k_k, z, \omega_c) G(k_k - k_a + k_c, \omega_c) G(k_k, \omega_c) \tag{74}$$

which represents the result of summing up the ladder series in the impurity potential. The crucial aspect of this calculation lies in this second appearance of a density correlation function, in addition to the density correlation mediated by the Bose fields. This arises from the fact that L_{ijkl} is simply related, by analytic continuation in frequency, to the ensemble averaged product of two large-N Green functions $G_{il}G_{kj}$. Fourier transforming the ensemble averaged expression for $\Sigma_E(i\omega)$ yields

$$\Sigma_E(i\omega_a) = \frac{1}{2\beta} \sum_{q, \omega_q} 2\pi N^*(0) \tau \mathrm{disc} S(q, z, \omega_a - \omega_c) \times [S^{-1}_{\lambda\lambda}(q, \omega_q) + 4t_0 \rho S^{-1}_{\lambda\rho}(q, \omega_q) + (2t_0\rho)^2 S^{-1}_{\lambda\lambda}(q, \omega_q)] \tag{75}$$

which, using the low-q, low-ω form of the Bose propagators, becomes

$$\Sigma_E(i\omega_a) = \frac{1}{2\beta} \sum_{q, \omega_q} 2\pi N^*(0) \tau N^*(0) \mathrm{disc}_z S(q, z, \omega_a - \omega_a) I(q, \omega_q) \tag{76}$$

where

$$I(q, \omega_q) = 4|t_0(q)|/[1 - 4|t_0(q)| f(q, \omega_q)] \tag{77}$$

and

$$t_0(q) = t_0 - Cq^2/8m\rho^2 \tag{78}$$

where $C = n_f$, the electron filling, and t_0 denotes the bare kinetic energy evaluated at the Fermi surface. Analytically continuing the low-wavevector, low-frequency expansion (21) for S leads, after some simplification, to the following expression for the self-energy:

$$\Sigma_E(i\omega_a) = \frac{N^*(0)}{2\beta}\sum_{q,\omega_q}\frac{I(q,\omega_q)}{i\omega_a - i\omega_q - E + iDq^2}$$

$$= 2\sum_q |t_0(q)|N^*(0)T\sum_{\omega_q}\frac{|\omega_q + Dq^2|}{[\omega_q + A(q)Dq^2](i\omega_a - i\omega_q - E + iDq^2)} \tag{79}$$

where

$$A(q) = 1 + 2NN^*(0)|t_0(q)|. \tag{80}$$

Converting the sum over ω_q into an integral in the usual manner yields, after the analytic continuation of $i\omega_a$ onto just above the real axis,

$$\Sigma_E(\omega + i\delta) = 2\sum_q |t_0(q)|N^*(0)\frac{1}{2\pi}\int_{-\infty}^{\infty} dx \left\{\frac{x\tanh(x/2T)[1 - A(q)]Dq^2}{x - \omega - i\delta + E - iDq^2}\right.$$

$$\left.\times\frac{1}{x^2 + (A(q)Dq^2)^2} - \frac{\coth(x/2T)Dq^2(\omega + x + iDq^2)}{[(x+E)^2 + (Dq^2)^2](\omega + x + iDA(q)q^2)}\right\}. \tag{81}$$

On taking the imaginary part of this expression we obtain the following expression for the scattering rate:

$$\mathrm{Im}\,\Sigma_E(\omega + i\delta) = \frac{1}{\pi}\sum_q |t_0(q)|N^*(0)\int_{-\infty}^{\infty} dx\left[\tanh\left(\frac{x}{2T}\right) - \coth\left(\frac{x-\omega}{2T}\right)\right]$$

$$\times\frac{x(Dq^2)^2[1 - A(q)]}{[x^2 + A(q)^2D^2q^4][(x - \omega + E)^2 + (Dq^2)^2]} \tag{82}$$

which, at zero temperature, after carrying out the x integral, becomes, for on shell states, such that $E = \omega$:

$$\mathrm{Im}\,\Sigma_{E=\omega}(\omega + i\delta) = \frac{N^*(0)}{2\pi}\sum_q\frac{|t_0(q)|}{1 + A(q)}\left[\ln\left(1 + \frac{\omega^2}{[A(q)Dq^2]^2}\right) - \ln\left(1 + \frac{\omega^2}{D^2q^4}\right)\right] \tag{83}$$

which we proceed to examine in the limits of moderate filling and close to $1/N$ filling.

4.1. Moderate filling

The low-energy behaviour is obtained by replacing t_0 and A by their low-wavevector limits, by carrying out the trivial angular q integration and by re-scaling the radial q variable by $\omega^{1/2}$ (or $\omega^{1/2}A$, whichever is appropriate), leading to

$$\mathrm{Im}\,\Sigma(\omega + i\delta) = \frac{S_d|t_0|N^*(0)}{2\pi[2 + 2NN^*(0)|t_0|]}\int_0^{\infty} dq\, q^{d-1}\ln\left(1 + \frac{1}{q^4}\right)D^{-d/2}[A^{-d/2} - 1]\omega^{d/2} \tag{84}$$

where S_d is the angular integration constant (2π in 2D, 4π in 3D). Comparing with the results of the non-disordered problem we can pick out a factor of the spin symmetric Landau parameter $A_0^s = 2NN^*(0)|t_0|/[1 + 2NN^*(0)|t_0|]$ that naturally arises in a study at the charge susceptibility of the $U = \infty$ Hubbard model in the large-N limit. The q integral is simply evaluated with the result that

$$\operatorname{Im}\Sigma(\omega + i\delta) = a_d D^{-d/2}[A^{-d/2} - 1]\omega^{d/2}A_0^s/(2 - A_0^s) \tag{85}$$

where $a_d = (1/4\pi N)S_d(\pi/d)(\sqrt{2})^{d-2}/(2\pi)^d$ for $d = 2$ or 3. Again, this power law behaviour can be compared with the very general 'Fermi liquid' results of Altshuler and Aronov [20] once we make the identification of the scattering rate

$$1/2\tau(\omega) = \operatorname{Im}\Sigma(\omega + i\delta) = (a_d/ND^{d/2})\lambda\omega^{d/2} \tag{86}$$

where $a_1 = \sqrt{2}/8\pi$, $a_2 = 1/16\pi$, $a_3 = \sqrt{2}/16\pi^2$ and $\lambda = 4F\{[1 - (F/2)^{d/2}] - 1\}/[d(4 - F)]$, in agreement with the functional form of the $j = 1$ result of Altshuler and Aronov [20] if we take $F = 2A_0^s$. Note that this agreement between the functional dependences of the two theories is to some extent superficial, since the present theory is concerned with density fluctuations rather than the $j = 1$ spin fluctuations for which the Altshuler and Aronov results [20] take the form of λ given above. This explains a missing factor of three, which arises from the spin multiplicity of the $j = 1$ spin fluctuations [20]. The density fluctuation contributions in [20] do not take such a functional form because the long-range Coulomb interaction swamps the short-range Fermi liquid parameters in this channel.

Nevertheless, the functional forms of the two types of theory are similar, with the present approach being able to derive the coefficients of the power laws directly in terms of the microscopic model parameters. The power laws in the two types of theory are the same away from the critical metal–insulator filling.

4.2. Near 1/N filling

In this case the q dependence of $t_0(q)$, $A(q)$ dominates over the residual ($q = 0$) constant values so that

$$\operatorname{Im}\Sigma(\omega + i\delta) = \frac{1}{4\pi N}\sum_q \left[\ln\left(1 + \frac{\omega^2}{(D'q^4)^4}\right) - \ln\left(1 + \frac{\omega^2}{D^2q^4}\right)\right] \tag{87}$$

and we have defined $D' = [2cNN^*(0)/8m\rho_0^2]D = \alpha D/k_F^2$, where $\alpha = (3n^2/4\pi^3)(m^*/m)^3$. Performing the same re-scaling operations as in the moderate-filling case, we obtain

$$\operatorname{Im}\Sigma(\omega + i\delta) = (S_d/4\pi N)[f(d, 8)(\omega/D')^{d/4} - f(d, 4)(\omega/D)^{d/2}] \tag{88}$$

where

$$f(d, n) = \int_0^\infty dq\, q^{d-1}\ln(1 + 1/q^n)/(2\pi)^d.$$

Thus, in addition to the conventional $\omega^{1/2}$ behaviour we obtain a sharper $\omega^{d/4}$ correction. Ultimately, for finite mass enhancements, the $\omega^{d/4}$ term will revert back to the $\omega^{d/2}$ component in (85), at energies of the order of the crossover scale $\varepsilon_F(\varepsilon_F\tau)^2A_0^2(m/m^*)^2$ discussed in the last section.

To conclude this section, we have extracted the on shell quasiparticle scattering rate for the disordered $U = \infty$ Hubbard model with the aid of the $1/N$ expansion. While for small to moderate fillings we obtain the $\omega^{d/2}$ law characteristic of weak-interaction theories, we obtain an additional $\omega^{d/4}$ component as the filling approaches its critical value, and the interactions become more important. While the extraction of such power law behaviour in experimental situations is likely to be difficult, these power laws do indicate that a qualitatively different description needs to be applied in order to understand the low-temperature behaviour of strongly interacting disordered systems. This result supports the contention established in the previous section that the combination of disorder and interaction tends to enhance the instabilities of the metal–insulator transition.

5. Summary and conclusions

In conclusion, we have, in this paper, attacked the outstanding problem concerning the interplay between electron correlation and disorder from a standpoint that (i) treats the strong correlations as foremost, and (ii) studies the effects of weak disorder. The strong-correlation effects are studied in the context of a version of the $U = \infty$ Hubbard model that allows a $1/N$ expansion to be carried out, using the now standard auxiliary boson technique, while the disorder effects are examined by summing up diffusive corrections in the random potential. These enter not only at the mean field level, where they slightly modify the large-N mean field equations, but more importantly at the Gaussian correction level $1/N$ where the diffusive ladder diagram corrections dress the RPA like graphs that determine the density correlation function, the specific heat corrections, and the on shell quasiparticle decay rate.

At fillings away from the critical metal–insulator value $n = 1/N$ the results are in qualitative agreement with earlier weak-interaction approaches. The density correlation function exhibits a diffusion pole, albeit with a modified diffusion constant, and this is reflected in power law corrections to the specific heat and decay rate of the disordered quasiparticle states that emerge at the mean field level. The Landau like parameters that enter into the earlier approaches are however, in our treatment, uniquely determined in terms of the microscopic model parameters, namely, for the model under study, the electron filling.

However, as the critical metal–insulator filling is approached the behaviour of the above quantities changes—as a result of the novel nature of the Gaussian fluctuations in the pure problem, where radial fluctuations in the Bose fields are associated with 'holon' motion [16–18], the formerly momentum independent Landau parameter that determines the physics away from the critical filling acquires a strong momentum dependence. This leads to a strong modification of the diffusion pole, and hence to sublinear power law dependences of the specific heat on temperature, and the on shell scattering rate on energy. While these corrections may be difficult to isolate in practice their existence from a theoretical viewpoint indicates that the strongly interacting electron problem is highly sensitive even to weak disorder. As the physical origin of these effects is not particularly clear, it would be of interest to try to understand how diffusion is affected by 'holon' motion from a more macroscopic viewpoint.

It would be of interest to extend the present calculation to include antiferromagnetic coupling (the t–J model) or to the Anderson lattice. For both of these problems the present formalism is straightforwardly extended, either to include more fields, as in the t–J model, or to more bands, as in the Anderson lattice. Work on these aspects is in progress.

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